Towards a Framework for Assessing Deformable Models in Medical Simulation

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Abstract. Computational techniques for the analysis of mechanical problems have recently moved from traditional engineering disciplines to biomedical simulations. Thus, the number of complex models describing the mechanical behavior of medical environments have increased these last years. While the development of advanced computational tools has led to interesting modeling algorithms, the relevances of these models are often criticized due to incomplete model verification and validation. The objective of this paper is to propose a framework and a methodology for assessing deformable models. This proposal aims at providing tools for testing the behavior of new modeling algorithms proposed in the context of medical simulation. Initial validation results comparing different modeling methods are reported as a first step towards a more complete validation framework and methodology.

1 Introduction

Accurate and interactive simulations of medical environment offer new alternatives and potential helpful tools for the realization of the physician gestures. Thus, deformable models can provide information on the global behavior of soft biological materials, even for locations where it may be difficult to obtain experimental data. In addition, ongoing improvements of computational power make it possible to use more complex models and produce more realistic representations of medical environment. While these motivations have been a driving force for the rapid growth of deformable models, they have also triggered the development of the field of interactive medical simulation. However, in both contexts, a certain level of validation must be established before physicians can use such simulations, whether it is for planning a complex procedure or for learning basic surgical skills. The overall objective of a validation process is to guarantee that: (i) the numerical approximation of the mathematical equations chosen for governing the model is acceptable and (ii) the model provides an accurate representation of the physical behavior of the problem of interest within a given computation time. Both assumptions need to be verified within an assessment of error in the model predictions and their achievement relies on a combination of methodologies and experimental data.

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A review on verification, validation and sensitivity studies has recently been proposed in the context of computational biomechanics [1]. In their paper, the authors present the concepts of verification and validation of biomechanical models and introduce a guide to realize such studies. In the context of medical simulation, only some papers have namely proposed solutions to test, compare and quantify the results of different modeling methods, in particular deformable models for soft biological material simulations. Alterovitz et al. [2] have suggested accuracy metrics and benchmarks for comparing different algorithms based on the Finite Element Method (FEM). Validations procedures for discrete approaches have also been introduced [3,4,5]. However these studies are mainly focused on the identification of parameter sets that optimize the accuracy of the discrete models. Real data can also been used as reference models and experiment results have already been presented in the context of medical simulation. Among them, the Truth Cube experiment [6] or experiments on cylinders [7] offer quantitative results, allowing the comparisons of modeling methods with real three-dimensional data. A comparison of FEM simulations with medical images have also been proposed in [8].

All these papers aim at providing reference solutions for either verifying the numerical behavior of models with analytical solutions or validating them against real data. However, the proposed experiments are rarely shared and a methodology based on a combination of a protocol and associated measurements has not been defined yet.

In this paper, a methodology and a framework for assessing deformable models are proposed. The methodology, introduced in Sect. 2, is a combination of analytical models and experimental reference objects that can test the ability of various algorithms to capture a particular deformable behavior. In addition, different metrics are proposed to quantitatively assess the accuracy of algorithms, as well as their computational efficiency. The proposed framework is based on an open source simulation environment where several algorithms are already implemented, thus making it a more consistent basis for comparing algorithms against each other and for validating them against reference models. Its use is illustrated with an example combining analytical solution, real data and different modeling methods in Sect. 3. An initial series of tests illustrate our proposal.

2 Validation Methodology and Framework

2.1 Verification and Validation Protocol

Based on the guide proposed in [1], the protocol for analyzing the performances of a deformable model can be decomposed in two main parts. The first part concerns the verification process of the modeling method. It aims at determining if the model implementation provides a correct description and a solution of the chosen modeling method theory. In this part of the protocol, the benchmarks used to analyze the performances of the model are mainly analytical solutions of well-known problems. Such comparisons have already been proposed in the literature, for example by [2]. In a second stage called validation, the ability of the already verified model to bring a correct simulation of a real world object has to be guaranteed. In this validation part, computational predictions are compared to experimental data as a gold standard.

For both parts of the verification and validation protocol, different types of errors can be identified. The first type concerns numerical errors introduced by computational solving of intractable mathematical equations, among those discretization or convergence errors are very common. This type of error is mainly identified trough the verification process. The second type of error can be called modeling error and is related to assumptions and approximations in the mathematical representation of the physical problem of interest. Such errors mainly come from geometry representation, boundary condition specifications, material properties or the choice of the governing constitutive equations. They can mainly be measured through the validation process.

2.2 Measurements and Metrics

In medical simulators, two types of objectives can be differentiated. A simulator can be dedicated either for a learning task or for the planning of a medical procedure. To validate a deformable model used for the simulation of a medical environment, different performances criteria have to be defined. In the context of medical simulation, we focus on two specific criteria : computational time and accuracy. These criteria allow the evaluation of both the interactivity and the precision offered by a modeling method.

To measure accuracy performances of the different modeling methods, two different types of metrics are proposed, depending on the type of available reference data. For both types, the error can be an absolute value, taking into account the displacement value or a relative value independent of the displacement of the simulated object.

The first type of error metric is used if reference data contain markers (the mesh of an analytical solution or solid markers inside a phantom for example). The metric proposed in this paper is a relative value called the relative energy norm error. This metric is commonly used in the FEM literature [9] and for algorithm comparisons [2]. Let $\mathbf{u}$ be a vector containing the displacement of each point of a discretization of the reference solution and $\hat{\mathbf{u}}$ be a vector containing the simulated displacements of each point of a model (like nodes on a FEM model for example). The relative energy norm error η is defined as:

$$\eta = \|e\|/\|\mathbf{u}\| \tag{1}$$

where $\|e\|$ is the energy norm for the error between the two displacements $\mathbf{u}$ and $\hat{\mathbf{u}}$: $\|e\| = \sqrt{(\hat{\mathbf{u}} - \mathbf{u})^T(\hat{\mathbf{u}} - \mathbf{u})}$.

If reference data do not contain any marker but give information about their global shape (curve, surface, etc), an other error metric based on a measurement of the distance between the reference and the simulated shapes has to be defined. Research works on image registration can provide good metrics. In this paper, we propose a simple distance as a first step for a metric framework.

The measured distance corresponds to the minimal distance between the simulated points sampled on the surface of the simulated model and the surface of the reference data along the normal. This distance can also be normalized by the simulated displacement for each point. The obtained value is realistic only for small errors between the simulated and the reference displacements. This second metric preferentially gives information on the surface of the simulated objects while the first metric provides measurements on the entire volume.

Computational efficiency is also an essential parameter to consider for the assessment of interactive medical simulations. When dealing with dynamic or kinematic systems, a first measure consists in determining if the algorithm can achieve true real-time computation, i.e. the computation time t_{comp} required for a given time step is less or equal to the time step dt used in the time integration scheme of the algorithm: $t_{comp} \leq dt$. Now, to guarantee interactivity, we also must verify that $dt \leq 1/F_c$ where F_c is a critical frequency (typically 25Hz when only visual feedback is considered, and hundreds of Hertz when haptic feedback is needed). With static or quasi-static equations, the real-time criterion $t_{comp} \leq dt$ is irrelevant as the simulation only consists of a sequence of equilibrium states which are independent of the time sampling. However, the second criterion remains necessary, even if defined differently as: $t_{comp} \leq 1/F_c$. Based on these criteria, the definition of a metric could be a combination of measures of these values, pondered by the simulation objectives. Of course, these criteria and metrics are not the only possible means of evaluating computational efficiency, as many factors influence the overall computation time of soft tissue deformation algorithms. Elements such as the integration scheme, the static or dynamic state of the resolution algorithm and furthermore the computer used to solve the simulations (with the use of a CPU or a GPU based algorithm for example) can lead to variations in computational performances. However, measuring such computational performance can only make sense if it is performed whithin a common framework, to ensure a better impartiality in the measurements, as they are often used comparatively against other algorithms or methods.

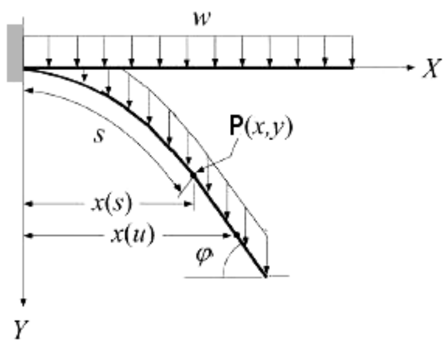
2.3 Validation Framework

In this paper, a framework is introduced in order to gather both reference models and metrics for assessing a given deformable model behavior. The chosen framework is an open source simulation environment SOFA where several algorithms are already implemented [10]. A validation environment was added to this framework, allowing to share different reference models (which are either analytical solutions or experimental data) and different solutions from existing modeling methods. The description language proposed in [11] to unify the description of the model and loads applied during a simulation is also used in this framework.

3 Example of a Validation Experiment

3.1 Description

In this paper, an example of comparison protocol is developed in order to illustrate how the metrics and the framework introduced in Sect. 2 can be used. The chosen experiment is an elastic beam under gravity, fixed on one side. This test case is widely known in continuum mechanics and has already been used previously for example by [12]. For this experiment, an analytical solution is available allowing for a verification procedure. Furthermore, real data experiments have been conducted in order to achieve a validation procedure. Doing so, different deformable solutions have been compared to these reference solutions.



L(m)	0.178
R(m)	0.019
ρ(kg.m^{-3})	1,070
E(Pa)	60,000
ν	0.49
g(m.s^{-2})	9.81

Fig. 1. Description of the beam experiment: definition and values of the parameters

Concerning the beam geometry, the beam cross-section is circular with a radius R and L is the length of the beam. Its density is called ρ, its Young Modulus E and its Poisson ratio ν. The experiment and parameter values are given in Fig. 1.

3.2 Analytical Solution

The Bernoulli-Euler theory of beams provides a formulation of the beam deflection, based on the assumption that a relationship between the bending moment M and the beam curvature κ exists: $\kappa = M/EI$ where I is the moment of inertia of the beam cross-section. For large deflections like in our experiment, the exact expression of the curvature is $\kappa = d\varphi/ds$ where s corresponds to the arc length between the fixed end and a point on the beam and $\varphi(s)$ is the slope of the beam at any point with respect to horizontal (see Fig. 1). If we differentiate the curvature expression with respect to s, we can obtain the differential equation that governs large deflections of the beam:

$$\frac{d^2\varphi}{ds^2} = \frac{1}{EI}\frac{dM}{ds} \tag{2}$$

The bending moment M at a point P of coordinates (x, y) for the deflected beam is given by: $M(s) = \int_s^L w(x(u) - x(s))du$ where $w = \rho g \Pi R^2$ represents the load corresponding to the gravity, uniformly distributed along the entire length. By differentiating this equation once with respect to s, taking into account the relation $cos\varphi = dx/ds$, and substituting it in Equation 2, we obtain the non-linear differential equation that governs the deflections of the beam under the gravity:

$$\frac{d^2\varphi}{ds^2} = -\frac{1}{EI} w(L - s) cos\varphi \tag{3}$$

The numerical solution of this problem was achieved using Maple software (Maple 11.0) and the solutions were compared to the different modeling methods.

3.3 Real Data

Our experimental reference model is a cylindrical beam made of silicone gel. To obtain a material with nearly linear elastic properties, we used a silicon rubber called ECOFLEX (Ecoflex0030).

The estimated Young modulus E is equal to $60,000$Pa and the Poisson ratio has a value of 0.49, as the material is considered as nearly incompressible. The beam was glued on one extremity to an inverted T-shaped vertical support made of plexiglas, and submitted to its own weight. It was then scanned in a helical CT scanner to produce of volume of $512 \times 512 \times 113$ voxels, with a voxel size of $0.425 \times 0.425 \times 0.62\text{mm}^3$. The shape of the deformed beam, illustrated in Fig. 2, was reconstructed using a Marching Cube algorithm to produce the reference surface and smoothed to remove noise artefacts.

3.4 Modeling Methods

Our framework allows to gather the analyical solution, the experimental data and different existing modeling methods. In this paper, five different algorithms were compared: (a) a linear FEM algorithm with a tetrahedral mesh, (b) a co-rotational FEM algorithm also with a tetrahedral mesh [13], (c) a co-rotational FEM algorithm with an hexahedral mesh, (d) an algorithm based on 6 Degrees of Freedom Beams [14], (e) a mass-spring network.

3.5 Results

The resulting simulations of different modeling methods available in the SOFA framework have been compared against both an analytical solution and experimental data. At the exception of the mass-spring model for which stiffness coefficients were adjusted to obtain the best behavior, the physical parameters used in all simulations correspond exactly to those measured on the experimental data. Concerning the comparisons with the analytical solution, the relative energy norm error can be used as the different meshes of the models and the analytical solution have the same number of nodes. The results are given in Fig. 3 and Table 1 for the position of tip of the beam. As for comparisons with

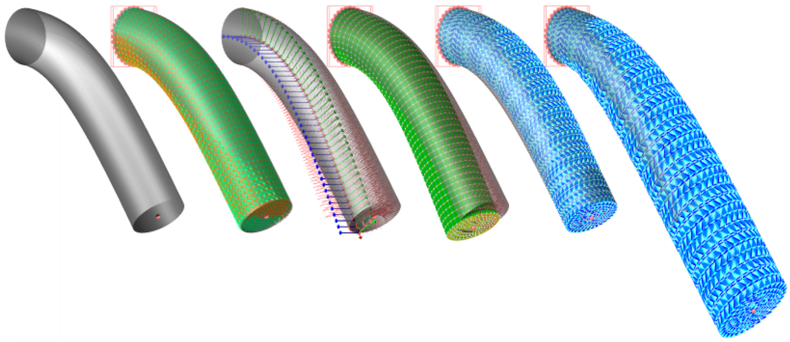


Fig. 2. Simulation results for the different modeling methods. From left to right: the experimental solution, the mass-spring network, the 6-DOF beam, the FEM Hexahedra, the FEM corotational Tetrahedra and the linear FEM Tetrahedra solutions.

Table 1. Comparisons between the different simulations: (a) the linear FEM tetrahedra algorithm, (b) the FEM Tetrahedra co-rotational algorithm, (c) the FEM hexahedra co-rotational algorithm, (d) the 6-DOF Beam algorithm and (e) the mass-spring network. The relative energy norm error is expressed in percentage and relies on the comparisons with the analytical solution. The surface error corresponds to the comparisons with the real data : the absolute value (without normalization) and the percentage are provided (the mean value and the standard deviation (SD)).

	Relative Energy Norm Error (%)	Relative Surface Error		
		Mean (mm)	Mean (%)	Standard Deviation
(a)	71.41	18.60	20.34	32.31
(b)	1.15	0.63	4.95	15.44
(c)	11.29	2.87	8.41	14.73
(d)	1.80	4.32	8.09	7.75
(e)	1.41	0.75	4.64	10.72

the experimental data, since no physical markers were used to track volumetric displacements, a measure of the relative surface error was used for the results reported in Table 1. Images of the simulated beams for the different algorithms are given in Fig. 2.

These results confirm, through quantitative measurements, important points about soft tissue modeling algorithms. First, if the underlying model is not appropriate, it is impossible to capture the deformation of the reference model, no matter the choice of parameters. This is well illustrated with the case of the linear elastic FEM model which cannot handle large displacements. On the other hand, our examples also show that it is possible to obtain rather good approximations of a given behavior using different methods (mass-spring model, co-rotational FEM, beam model) all within a range of computation times compatible with interactive simulations. We can also see that even an ideal, analytical model will not give the exact same result as an experiment, some of these differences coming from errors on the various measurements done on the experimental model. Our preliminary results also show the need for a variety of reference models, able to characterize various aspects of soft tissues, to clearly determine which algorithm is best for representing a particular behavior (linear elastic, visco-elastic, bi-phasic, porous, etc...). Similarly, it is important to define metrics that are most relevant to which property of an algorithm we want to evaluate. We have proposed an initial set of metrics to assess the accuracy of the models through comparisons with two different types of reference models. Additional metrics could certainly be proposed, in particular to evaluate the computational efficiency of a particular algorithm.

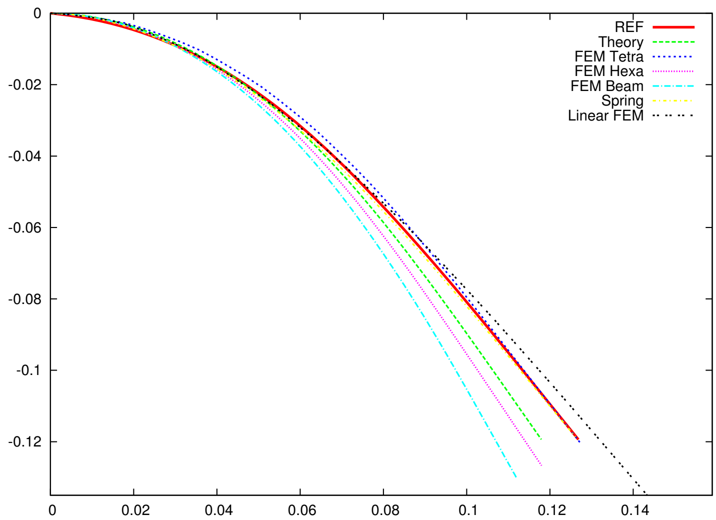


Fig. 3. Central line positions in Cartesian coordinates for each simulated beam, the analytical solution (Theory) and the experimental data (REF)

4 Conclusion

In this paper, we have introduced the basis of a methodology and framework for assessing deformable models in the context of medical simulation. The proposed methodology differs from existing protocols as medical simulators need to be validated both in terms of accuracy and computational efficiency. Although the set of metrics and reference models presented in this paper is limited, we believe they illustrate well the importance to quantitatively assess algorithms used in medical simulation. However, the main novelty of our approach lies in the combination of a unified, open framework where all models could be compared, new metrics defined, algorithms and reference models added. This will eventually enable an unbiased comparison of the performance and accuracy of many different algorithms, to create an Open Benchmark for medical simulation. To this end, we have planned to release the data, models and algorithms used in this study as part of the next SOFA release.

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